

Integration
by Substitution
التكامل بالتعويض

[I] $\sqrt{ax+b}$ $\sqrt[3]{a+bx}$ - - - -

Put $\sqrt{ax+b} = u \Rightarrow ax+b = u^2$

" $\sqrt[3]{ax+b} = u \Rightarrow ax+b = u^3$

EX: ① $\int \frac{1}{\sqrt{2x-1} + \sqrt[4]{2x-1}} dx = I$

Put $2x-1 = u^4$

$2dx = 4u^3 du \Rightarrow dx = 2u^3 du$

$I = \int \frac{1}{\sqrt{u^4} + \sqrt[4]{u^4}} \cdot 2u^3 du$

$= 2 \int \frac{u^3}{u^2 + u} du = 2 \int \frac{u^2 - 1 + 1}{u + 1} du = 2 \int \frac{(u-1)(u+1) + 1}{u+1} + \frac{1}{u+1}$

$= 2 \int u - 1 + \frac{1}{u+1} = 2 \left[\frac{u^2}{2} - u + \ln|u+1| \right] + C$

$$[2] \int \frac{1+\sqrt{x}}{1+\sqrt[3]{x}} dx$$

put $x = u^6$

$$dx = 6u^5 du$$

$$I = \int \frac{1+u^3}{1+u^2} \cdot 6u^5 du$$

$$= 6 \int \frac{u^5 + u^8}{1+u^2} du$$

↓
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$$= 6 \int u^6 - u^4 + u^2 - u - 1 + \frac{u+1}{u^2+1} du$$

$$= 6 \left[\frac{u^7}{7} - \frac{u^5}{5} + \frac{u^4}{4} + \frac{u^3}{3} - \frac{u^2}{2} - u + \frac{1}{2} \frac{u}{u^2+1} + \frac{1}{u^2+1} \right]$$

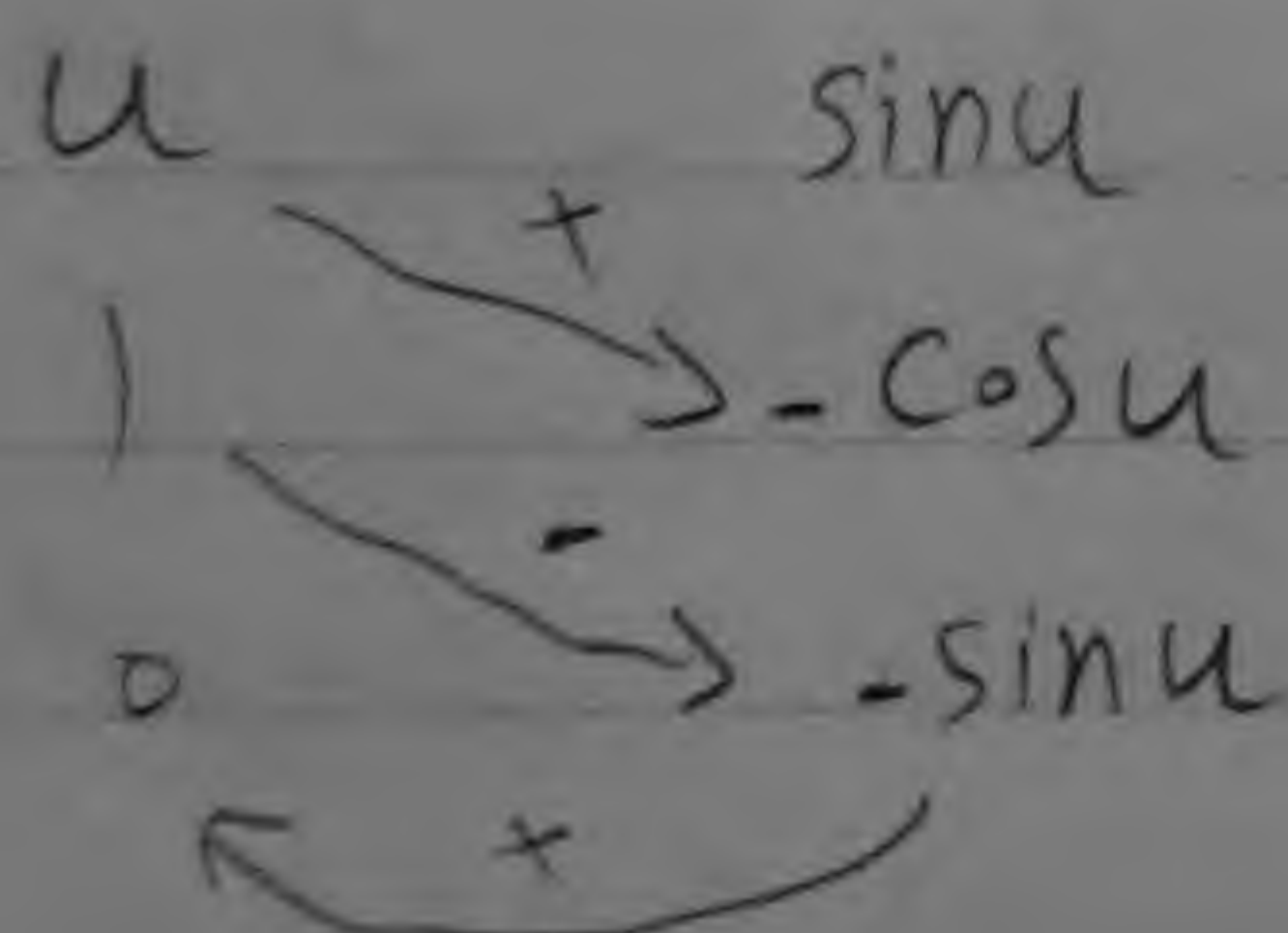
$$\begin{array}{r} \frac{1}{2} \ln|u^2+1| \quad \tan^{-1} u \\ \hline -u^6 + u^5 \\ + u^6 - u^4 \\ \hline u^5 + u^4 \\ - u^5 - u^3 \\ \hline u^4 - u^3 \\ - u^4 + u^2 \\ \hline -u^3 - u^2 \\ + u^3 + u \\ \hline -u^2 + u \\ + u^2 - 1 \\ \hline u + 1 \end{array}$$

$$\boxed{3} \int \sin(\sqrt{x}) dx$$

$$\text{Put } \sqrt{x} = u \rightarrow x = u^2$$
$$dx = 2u du$$

$$I = \int \sin(u) \cdot 2u du$$

$$= 2 \int u \sin(u) du$$



A diagram illustrating the integration by parts process. On the left, the terms 'u' and '1' are listed vertically. On the right, 'sin u' and '-cos u' are listed vertically. An arrow points from 'u' to '-cos u' with a '+' sign above it. Another arrow points from '1' to '-sin u' with a '-' sign above it. A curved arrow at the bottom points from '-sin u' back to 'u' with a '+' sign above it.

$$I = -u \cos u + \sin u + C$$

$$u = \sqrt{x}$$

$$[4] \int \frac{\cos \sqrt{x+1}}{\sqrt{x+1}} dx$$

$$\text{Put } \sqrt{x+1} = u \Rightarrow x+1 = u^2 \\ dx = 2u du$$

$$I = \int \frac{\cos u}{u} 2u du$$

$$= 2 \int \cos u du$$

$$= 2 \sin u + C$$

$$= 2 \sin \sqrt{x+1} + C$$

$$\boxed{5} \int \frac{e^x}{1+e^{2x}} dx$$

$$u = e^x$$

$$du = e^x dx$$

$$\int \frac{du}{1+u^2} = \tan^{-1} u + C$$
$$= \tan^{-1}(e^x) + C$$

$$\boxed{6} \int \frac{\sin(2\ln x)}{x} dx$$

$$u = 2\ln x$$

$$du = \frac{2}{x} dx$$

$$I = \frac{1}{2} \int \frac{2 \sin(2\ln x)}{x} dx$$

$$= \frac{1}{2} \int \sin u \, du$$

$$= -\frac{1}{2} \cos u + C$$

$$= -\frac{1}{2} \cos 2\ln x + C$$

نكالت على الصور *

$$[1] \int \sqrt{a^2 - x^2} dx \Rightarrow x = a \sin(\theta)$$

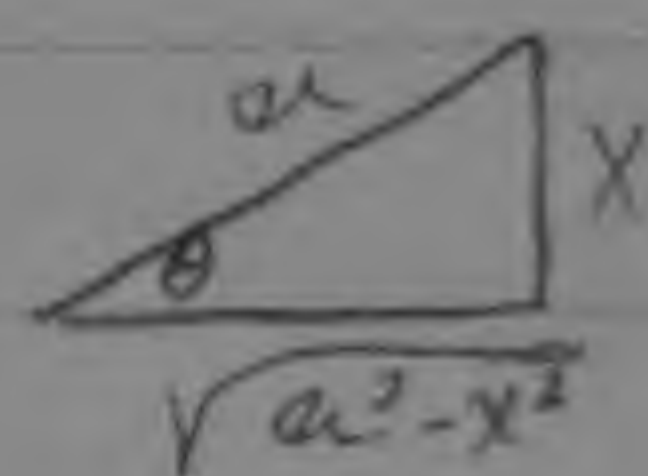
$$[2] \int \sqrt{x^2 - a^2} dx \Rightarrow x = a \sec(\theta)$$

$$[3] \int \sqrt{x^2 + a^2} dx \Rightarrow x = a \tan(\theta)$$

$$[1] \int \sqrt{a^2 - x^2} dx$$

put $x = a \sin \theta$

$$dx = a \cos \theta d\theta$$



$$\begin{aligned} I &= \int \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a \cos \theta d\theta \\ &= a^2 \int \sqrt{1 - \sin^2 \theta} \cdot \cos \theta d\theta \end{aligned}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \frac{x}{a} \cdot \frac{\sqrt{a^2 - x^2}}{a}$$

$$= \frac{2x \sqrt{a^2 - x^2}}{a^2}$$

$$= a^2 \int \cos \theta \cdot \cos \theta d\theta = a^2 \int \cos^2 \theta d\theta$$

$$= a^2 \int \frac{1}{2} (1 + \cos 2\theta) d\theta = \frac{a^2}{2} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{a^2}{2} \left[\theta + \frac{\sin 2\theta}{2} \right] + C, \quad \theta = \sin^{-1} \frac{x}{a}$$

$$= \frac{a^2}{2} \left[\sin^{-1} \frac{x}{a} + \frac{2x \sqrt{a^2 - x^2}}{2a^2} \right] + C$$